The inventory value of information sharing, continuous replenishment, and vendor-managed inventory

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Abstract

In this paper, we extend the models in the literature [Lee, H.L., So, K.C., Tang, C., 2000. The value of information sharing in a two-level supply chain. Management Science 46 (5), 626–643; Raghunathan, S., Yeh, A.B., 2001. Beyond EDI: impact of continuous replenishment program (CRP) between a manufacturer and its retailers. Information Systems Research 12 (4), 406–419] to analyze the benefits realized for manufacturers and retailers under information sharing (IS), continuous replenishment programs (CRP) or vendor managed inventory (VMI) and compare the distribution of benefits between manufacturers and retailers. Our analysis shows that IS, CRP, and VMI bring varying benefits in terms of inventory cost savings to firms, and that the benefits are not consistently distributed between retailers and manufacturers. Our findings also point to the managerial implications on how managers may decide the product sets and replenishment frequency for improved benefit realization under CRP and VMI.

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“...the chain has always managed its inventory well, but we want to take things to a higher level...”

James Sinegal, CEO of Costco, commenting on Costco’s VMI practice with Kimberly-Clark.

1. Introduction

During the past several years, a number of sophisticated supply chain management initiatives have been developed that make use of evolving information technologies (Emigh, 1999). These initiatives include collaborative planning, forecasting, and replenishment (CPFR), vendor-managed inventory (VMI), continuous
replenishment programs (CRP), and efficient consumer response (ECR). The result has been changes in industrial structures and improvements in firm performance. Firms have been able to reengineer their supply chains through real-time information sharing, enabled by electronic commerce technologies, such as electronic data interchange (EDI) and the Internet. Using CRP, buyers and suppliers share inventory status information so that they can increase replenishment frequencies and reduce inventory for both firms. With VMI, suppliers are authorized to manage inventories at buyers’ locations and can rationalize inventory in the supply chain.

Information sharing (IS) is a collaborative program in which the downstream firm (referred to as a retailer herein) agrees to provide demand and inventory status in real time to the upstream firm (referred to as a manufacturer herein) (Lee et al., 2000). In this case, the manufacturer no longer observes consumer demand through the retailer’s order quantities but determines it directly from end consumers, though the manufacturer still receives orders from the retailer (i.e., the retailer is responsible for placing orders). Both CRP and VMI provide closer collaboration between the manufacturer and the retailer. In addition to information sharing, CRP requires the manufacturer to implement a continuous replenishment process with the retailer; that is, increase the frequency of replenishments. In contrast, VMI is defined as collaboration between a manufacturer and a retailer, such that the manufacturer is authorized to manage the inventory at retail locations (in addition to information sharing and more frequent replenishments). The difference between CRP and VMI is that with VMI, the retailer no longer places orders with the manufacturer, but the manufacturer makes ordering decisions on behalf of the retailer on the basis of the shared information received from the retailer. Fig. 1 presents a descriptive comparison of information flows and physical goods flows among the baseline system (i.e., no information sharing) and the three collaborative systems.

These definitions are consistent with those used in industry. For example, an industry white paper (http://www.vendormanagedinventory.com/crp.pdf; accessed on 2/5/2006) describes CRP as, “an efficient replenishment concept within the Efficient Consumer Response (ECR) arena. It focuses on improving the flow of products in the supply chain, both forward to the customer and eventually the end consumer, and backward to the supplier”. It further states that VMI is “a term widely used when implementing CRP”, in which case inventory decisions shift from the buyer to the supplier.

Information sharing is generally facilitated through the use of either electronic data interchange (EDI) or the Internet. Shared information includes point-of-sale, demand, and inventory information among firms in the supply chain. For example, in EDI-enabled IS, CRP, or VMI, there are two EDI transactions at the heart of the information exchange process. The first is the Product Activity Record, which is often referred to by the Uniform Code Council (UCC) standard as UCS 852. UCS 852 contains sales and inventory information segmented into various classifications, such as inventory-on-hand, inventory-on-order, committed inventory, and back orders. UCS 852 is sent by the retailer to the manufacturer on a prearranged schedule, typically daily. The other transaction is the Purchase Order Acknowledgement sent from the manufacturer to the retailer on a prearranged schedule, typically daily. The other transaction is the Purchase Order Acknowledgement sent from the manufacturer to the retailer, or UCS 855. UCS 855 contains the product numbers and quantities ordered by the manufacturer on the retailer’s behalf.
The potential benefits from programs such as CRP and VMI are compelling, encompassing reduced inventory costs and improved customer service (Achabal et al., 2000; Waller et al., 1999). These benefits have been realized by successful retailers, most notably Wal-Mart (Cetinkaya and Lee, 2000). Research conducted on the benefits of these supply chain initiatives include Lee et al. (2000), who find that in a two-level supply chain the value of IS can be quite high when demands are significantly autocorrelated. Other key research includes Raghunathan and Yeh (2001), who compare the values of IS and CRP and conclude that continuous replenishment provides additional benefits over and above those derived from information sharing alone (i.e., without added CRP).

In this paper, we extend the work of Lee et al. (2000), hereafter referred to as LST, and Raghunathan and Yeh (2001), hereafter referred to as RY, to compare the benefits of IS, CRP, and VMI directly for both retailers and manufacturers. We quantify the value of, and benefit distributions from, these three types of supply chain collaborations. In particular, we make the following original contributions:

- Articulate the differences in benefits and the distribution of benefits among IS, CRP, and VMI. In the literature, CRP and VMI are often used interchangeably. However, the differences between the two processes may affect the realization and distribution of benefits.
- Quantify the value of VMI. Although LST studied the value of IS and RY studied the value of CRP, the value of VMI has not been well explored.
- Identify the parameters that affect the magnitude and distribution of benefits derived from CRP and VMI.

The remainder of this paper is organized as follows: Section 2 reviews relevant literature; Section 3 constructs an analytical model to quantify the value and value distribution of IS, CRP, and VMI; Section 4 presents a discussion of the findings from the analytical model, numerical examples, and managerial implications; and finally, Section 5 concludes the paper.

2. Literature review

Three streams of literature are relevant to our study: literature on (1) interorganizational systems (IOSs); (2) quantitative models in information sharing; and (3) research on IS, CRP, and VMI. In theory, supply chain management initiatives, such as IS, CRP, and VMI, can be categorized as IOSs. Research has shown that IOSs, as links between suppliers and buyers, can improve a firm’s performance and bring it competitive advantages (Bakos, 1991; Cash and Konsynski, 1985; Clemons and McFarlan, 1986; Palmer and Markus, 2000; Premkumar and Ramamurthy, 1995; Sethi et al., 1993; Srinivasan et al., 1994). Specifically, in conjunction with supply chain management, studies show that reduced inventory represents one of the major benefits of the implementation of IOSs, either enabled by EDI or through Internet technologies (Choudhury et al., 1998; Daugherty et al., 1999; Dresner et al., 2001; Lee and Whang, 2000; Mukhopadhyay et al., 1995; Premkumar, 2000; Raghunathan, 1999; Stank et al., 1999; Strader et al., 1999; Waller et al., 1999).

A second stream of research has examined quantitatively the value of information sharing in supply chains. The consequences of the bullwhip effect, for example, can be minimized through information sharing (Cachon and Fisher, 2000; Chen et al., 2000; Gavirneni and Kapuscinski, 1999; Lee et al., 1997; LST). Other research has found that policies such as VMI can decrease the bullwhip effect, thereby improving supply chain efficiency, such as by lowering inventory levels and reducing cycle time (Cachon and Zipkin, 1999; Corbett, 2001; Kulp et al., 2004; Mishra and Raghunathan, 2004; Xu et al., 2001). Particularly, LST study the value of information sharing in a two-level supply chain and find that the value of sharing demand information can be quite high, particularly when demands are significantly correlated over time. Raghunathan (2001), however, points out that the value of information sharing can be insignificant if the manufacturer uses the order history to forecast the retailer order quantity. Angulo et al. (2004), using simulation, find demand information sharing is a significant part of VMI implementation and can improve the fill rate by as much as 42%.

A number of research papers have studied the value of CRP, VMI, and other supply chain programs. Yang et al. (2003), using simulation, examine a VMI distribution system and conclude that VMI is very effective at mitigating the bullwhip effect. Waller et al. (1999) study VMI in retail supply chains also using simulations and find the operational benefits of VMI are very compelling, especially in reducing inventory without
compromising customer service levels. Daugherty et al. (1999) examine the automatic replenishment programs (ARP) enabled by EDI technologies and find overall performance to be positively associated with ARP effectiveness. Stank et al. (1999) study interfirm supply chain coordination in the food industry, most notably ECR, and conclude it leads to decreased inventory levels, decreased order cycle time, and decreased order cycle variance. Cetinkaya and Lee (2000) present an analytical model for coordinating inventory and transportation decisions with VMI systems. They find that the actual inventory requirement at the vendors is partly determined by the parameters of the shipment release policy. This result holds because vendors have the autonomy to keep small orders until an agreeable dispatch time is reached, with the expectation that an economical, consolidated dispatch quantity accumulates. In modeling CRP between a manufacturer and its N retailers, RY analyze the impact of information sharing and continuous replenishment in the CRP context and find that the value of CRP is affected by demand characteristics, such as its variance. In a similar approach, Aviv (2002) compares VMI with information sharing and with collaborative forecasting/auto replenishment (CFAR) and shows that VMI and CFAR are better at reducing inventories as autocorrelation in demand increases. Empirical evidence presented by Lee et al. (1999) confirms the value of supply chain coordination. These authors show that inventory turns and stockouts improved after the implementation of CRP, using data collected from 31 grocery retail chains.

The literature clearly recognizes that inventory reduction can be reached by implementing supply chain coordination initiatives, such as IS, CRP, or VMI. This paper contributes to the literature by analyzing IS, CRP, and VMI models to provide a better understanding of the extent of inventory cost savings and the distribution of these savings between participants using these supply chain programs.

3. Analytical model

3.1. Modeling framework

We consider a simple, two-level supply chain that consists of one manufacturer and one retailer and examine inventory management practices before and after they implement IS, CRP, and VMI systems. We assume a single product is transacted between the manufacturer and the retailer. The retailer faces external demand from end consumers. Any orders for the retailer that are not fulfilled immediately due to stockouts are back-ordered with penalty cost. The retailer decides and places orders with the manufacturer under IS and CRP but not VMI. (Under VMI, the manufacturer makes these decisions.) The manufacturer implements a build-to-stock product policy and ships the required orders to the retailer from inventory once the orders are received. If the manufacturer does not have enough stock to fill the retailer order, the manufacturer expedites production with its incurred costs to meet the shortfall (as an alternative, the manufacturer may use another supply source at a premium price to meet the shortfall).

Similar to LST, we consider an order-up-to-level periodic review system \((R, S)\). At every \(R\) units of time; that is, at each review instant, sufficient products are ordered to raise the inventory position at the retailer to level \(S\). To simplify the analysis, without loss of generality, we make the inventory reviewing cycle time 1, so \(R = 1\). (The 1 unit of time is still meaningfully divisible; e.g., 1 week can be divided into seven 1-day segments.) We further assume stochastic demand of AR(1), which has been widely used to model demand processes in the literature (LST; RY; Aviv, 2002). AR(1) is a first-order autoregressive process that is structured so that the influence of a given disturbance fades as it recedes into the more distant past but vanishes only asymptotically (Greene, 1997). The mathematical form of the AR(1) process is defined as follows:

\[
D_t = d + \rho D_{t-1} + \epsilon_t,
\]

where \(d > 0\), \(-1 < \rho < 1\), and \(\epsilon_t\) is i.i.d. normally distributed with mean zero and variance \(\sigma^2\).

Let \(H\) and \(h\) denote the unit inventory holding costs per period for the manufacturer and the retailer, respectively. The inventory holding costs are stationary over time. Let \(P\) and \(p\) denote the penalty costs associated with the manufacturer’s expedited production and the retailer’s back orders, respectively. Let \(K\) and \(k\) denote the service level requirements for the manufacturer and the retailer, respectively. Let \(L\) and \(l\) denote the replenishment lead time for the manufacturer and retailer, respectively. Following the notion in LST, we also
3.2. Ordering decisions without collaboration and with IS

As shown by LST, the manufacturer’s average inventories in the cases of no information sharing, $I_m$ (the base case), and information sharing, $I_m^{IS}$, can be approximated as follows:

$$I_m = \frac{d}{2(1-\rho)} + K\sigma\sqrt{V}, \quad \text{and}$$

$$I_m^{IS} = \frac{d}{2(1-\rho)} + K\sigma\sqrt{V'},$$

where:

$$V = \frac{1}{(1-\rho)^2} \left\{ (1-\rho)^2 + \sum_{i=1}^{L} (1-\rho^{L+L+3-i})^2 + \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{L+1})^2}{(1-\rho)^2} \right\},$$

and

$$V' = \frac{1}{(1-\rho)^2} \left\{ (1-\rho)^2 + \sum_{i=1}^{L} (1-\rho^{L+L+3-i})^2 \right\}.$$

In Eqs. (2) and (3), the first terms are the average cycle inventory, and the second terms are the average safety stock satisfying the service level $K$ for the manufacturer. The safety stock is affected by the service level requirement ($K$), residual demand variance ($\sigma$), and factors $V$ and $V'$. The implementation of IS has no impact on the cycle inventory but does impact safety stock. As derived by LST, $V'$ is always less than $V$, so information sharing reduces the manufacturer’s safety stock, thereby reducing the average inventory level. The retailer’s inventory level remains unchanged with information sharing, because the retailer does not alter its operations except for providing the manufacturer with demand information. In other words, the collaboration of IS will not bring any benefits in terms of inventory cost savings to the retailer. Using the same approach for the retailer, we can approximate the retailer’s average inventory when time $t$ approaches positive infinity as follows:

$$I_t = I_t^{IS} = \frac{d}{2(1-\rho)} + k\sigma\sqrt{v},$$

where:

$$v = \frac{1}{(1-\rho)^2} \sum_{j=1}^{t+1} (1-\rho^j)^2.$$

To compare the benefits and benefit distribution with other kinds of collaboration, we introduce three variables to measure collaboration: benefits to the manufacturer ($\Delta I_m$), benefits to the retailer ($\Delta I_t$), and the benefit distribution ratio ($G_{\%}$). Benefits to the manufacturer and benefits to the retailer are the inventory reductions for the manufacturer and the retailer, respectively, in moving from the base case of no information sharing to information sharing. The distribution ratio measures how the benefits from a supply chain program are distributed between the manufacturer and the retailer and is calculated using benefits to the manufacturer in percentage terms divided by benefits to the retailer in percentage terms. An immediate intuition from the distribution ratio is that the benefits are equally distributed between the manufacturer and the retailer when $G_{\%} = 1$; distributed in a greater percentage to the manufacturer when $G_{\%} > 1$; and distributed in a smaller percentage to the manufacturer when $G_{\%} < 1$.

According to the proceeding model, in moving from the base case to information sharing, the manufacturer’s and the retailer’s inventory levels change as follows:

$$\Delta I_m^{IS} = I_m - I_m^{IS} = K\sigma\left( \sqrt{V} - \sqrt{V'} \right),$$

$$\Delta I_t^{IS} = I_t - I_t^{IS} = 0.$$
The distribution ratio can be computed as:

\[
G^S_r = \frac{\Delta I^S_m/I_m}{\Delta I^S_r/I_r} = \infty.
\]  

(10)

### 3.3. Ordering decision with CRP

We define CRP as a higher level of collaboration between the manufacturer and the retailer. The manufacturer not only shares demand information with the retailer, but also implements a program to increase replenishment frequencies (instead of replenishing only once at the beginning of each period). (Theoretically, the replenishment frequency can be increased to positive infinity; that is a “true” continuous replenishment process.)

As RY show, the benefits of CRP to the manufacturer and retailer in terms of reductions of their inventory levels from the base case, respectively, are the following:

\[
\Delta I^C_m = \frac{d}{2(1-\rho)} + K\sigma\left(\sqrt{V} - \sqrt{V'}\right), \quad \text{and}
\]

\[
\Delta I^C_r = \frac{d}{2(1-\rho)} + k\sigma\sqrt{v}. \tag{11}
\]

In RY’s paper, it is assumed that the retailer’s inventory drops to a negligible level (zero) after CRP implementation, because the benefits of CRP to the retailer are not the focus of the paper. However, this assumption may not be the case. In the base case, the retailer’s average inventory level is \(\frac{d}{2(1-\rho)} + k\sigma\sqrt{v}\). The first element in the expression is the cycle inventory, whereas the second element is the safety stock that satisfies the service level of \(k\). The safety stock is determined by the variation in demand, desired level of service, and replenishment lead time (i.e., the time between when an order is placed and when an order is received). If the retailer’s inventory is assumed to be zero, the service level cannot be maintained at \(k\).

To better compare the base case to CRP, we assume that the retailer will maintain the same service level. We consider a CRP case presented in RY, in which the inventory review period does not change, but the replenishment frequency increases to \(g\) times per review period, as opposed to 1 time per review period. For example, we may consider a situation in which a manufacturer continuously produces a retailer’s order and ships a truckload at the end of every week, as opposed to accumulating inventory until the end of month and shipping the four truckloads at one time. (For simplicity’s sake, we assume 4 weeks per month.) Because the replenishment lead time does not change from the base case, the level of safety stock required to maintain the same service level does not change either. Thus, the benefit (i.e., inventory reduction) of CRP to the retailer is only a reduction in cycle stock and can be written as:

\[
\Delta I^C_r = \frac{d}{2(1-\rho)} + k\sigma\sqrt{v}. \tag{12}
\]

Assume there are \(g\) replenishments during a review period. For AR(1) demand with \(g\) divisible periods, each divisible period also follows AR(1), and the average inventory level for each \(g\) divisible period within a review period is just \(1/g\) times the average demand for the whole review period (see Lemmas 1 and 2 for a detailed proof in the Appendix). The cycle inventory for a \(1/g\) period can be calculated as \(1/g\) times the original cycle inventory. Because the demand structure, service level requirements, replenishment lead time, and, most important, the review period remain the same, the safety stock level for the retailer should also be unchanged. As indicated previously, however, the retailer’s cycle stock will decrease. With shipments received \(g\) times per review period, inventory accumulates gradually and evenly throughout the review period, with every subdivided period bringing the cycle inventory up to just \(1/g\) times the order-up-to level. Hence, the average retailer’s inventory for the \(i\)th \(1/g\) period can be expressed as:

\[
I^C_{ri} = \frac{1}{g} \frac{d}{2(1-\rho)} + k\sigma\sqrt{v}. \tag{13}
\]

The overall average retailer’s inventory level for a period \(t\) then can be computed as:
\[ r_{\text{rg}}^{\text{CRP}} = \frac{1}{g} \sum_{i=1}^{g} I_{ni} = \frac{1}{g} \frac{d}{2(1 - \rho)} + k\sigma \sqrt{v}. \]  
\[ (15) \]

Similar to the retailer, the manufacturer’s average inventory level with \( g \) replenishments during the review period can be expressed as:

\[ r_{\text{mg}}^{\text{CRP}} = \frac{1}{g} \frac{d}{2(1 - \rho)} + K\sigma \sqrt{\bar{V}}. \]  
\[ (16) \]

Note that when \( g \) equals 1, the results are the same as with IS (i.e., IS is a special case of CRP with \( g = 1 \)), whereas when \( g \) approaches positive infinity, the replenishment interval is infinitely small, so the results approximate a true “continuous” replenishment program.

We can compute the benefits (as also shown in RY) and the benefit distribution ratio for CRP as follows:

\[ \Delta l_{\text{m}}^{\text{CRP}} = I_{\text{m}} - r_{\text{mg}}^{\text{CRP}} = \frac{g - 1}{g} \frac{d}{2(1 - \rho)} + K\sigma \left( \sqrt{\bar{V}} - \sqrt{\bar{V'}} \right), \]  
\[ (17) \]

\[ \Delta l_{\text{r}}^{\text{CRP}} = r_{\text{r}} - I_{\text{r}}^{\text{CRP}} = \frac{g - 1}{g} \frac{d}{2(1 - \rho)}, \]  
\[ (18) \]

\[ G_{\text{mg}}^{\text{CRP}} = \frac{\Delta l_{\text{m}}^{\text{CRP}}}{I_{\text{m}}} = \left( \frac{g - 1}{g} \frac{d}{2} + 2g(1 - \rho)K\sigma \left( \sqrt{\bar{V}} - \sqrt{\bar{V'}} \right) \right) \left[ \frac{d + 2(1 - \rho)K\sigma \sqrt{\bar{V}}}{d + 2(1 - \rho)K\sigma \sqrt{\bar{V'}}} \right]. \]  
\[ (19) \]

3.4. Ordering decision with VMI

In this section, we analyze the case in which VMI is implemented between the manufacturer and the retailer. We assume that with VMI, the manufacturer manages the retailer’s inventory, so the retailer no longer needs to place orders to the manufacturer. The manufacturer orders from its upstream suppliers on the basis of end-consumer demand and strategically distributes inventory between the manufacturer’s and the retailer’s locations.

Similar to CRP, we consider the case in which there is no change in the inventory review period, but replenishment frequency changes to \( g \) times per review period. We now compute the manufacturer’s order-up-to levels and average inventory level for VMI with one replenishment per period, and then discuss the general case of \( g \) replenishments per review period. In the (1, S) periodic inventory review system, after an order has been placed at time \( t \), no other later orders can be received until time \( t + L + 1 \). Therefore, the order-up-to level at time \( t \) must be sufficient to cover demand for the duration of lead time plus the review period, i.e., \( L + 1 \). Thus, we need to know the demand over the manufacturer’s lead time and a review period (i.e. \( L + 1 \)) to determine the order-up-to level. By using the recursive relationship of \( D_t \) in AR(1), we can obtain the total demand quantity over the manufacturer’s leadtime (\( L \)):

\[ \sum_{t=1}^{L+1} D_{t+i} = \frac{1}{1 - \rho} \left[ d \sum_{t=1}^{L+1} (1 - \rho^i) + \rho(1 - \rho^{L+1})D_t \right] + \varepsilon_{t+L+1} + (1 + \rho)\varepsilon_{t+L} + \cdots + (1 + \rho + \rho^2 + \cdots + \rho^L)\varepsilon_{t+1}. \]  
\[ (20) \]

Because information sharing exists between the manufacturer and the retailer, such that \( D_t \) and \( \varepsilon_t \) are common knowledge for the manufacturer, the conditional mean and conditional variance of the total demand quantity over the manufacturer’s lead time, given current demand and variation, are as follows:

\[ m_t = E \left( \sum_{i=1}^{L+1} D_{t+i} \right) = \frac{d}{1 - \rho} \left[ \frac{(L + 1) - \sum_{i=1}^{L+1} \rho^i}{1 - \rho} \right] + \rho(1 - \rho^{L+1})D_t, \]  
\[ (21) \]

\[ v_t' = \text{Var} \left( \sum_{i=1}^{L+1} D_{t+i} \right) = \left[ \frac{1}{(1 - \rho)^2} \sum_{i=1}^{L+1} (1 - \rho^i)^2 \right] \sigma^2, \]  
\[ (22) \]

where, \( v' = \frac{1}{(1 - \rho)^2} \sum_{i=1}^{L+1} (1 - \rho^i)^2 \).
Hence, the order-up-to level can be expressed as $T_t = m_t + K\sigma \sqrt{v}$. Following the same approximation of the average on-hand inventory as shown in Silver and Peterson (1985), the average inventory for the manufacturer can be obtained from:

$$I_{m}^{VMI} = T_t - E\left(\sum_{i=1}^{l+1} D_{t+i}\right) + \frac{E(D_t)}{2},$$

where $T_t$ is the order-up-to level, $E\left(\sum_{i=1}^{l+1} D_{t+i}\right)$ is the forecasted demand over a review interval plus a replenishment lead time in units, and $E(D_t)$ is the expected demand rate in units per period.

Therefore, average inventory for the manufacturer under VMI can be calculated as:

$$I_{m}^{VMI} = \frac{d}{2(1 - \rho)} + K\sigma \sqrt{v}.$$  

To implement a VMI program, the retailer likely will insist that its service level is maintained at pre-VMI levels (e.g., Fry et al., 2001). Therefore, we assume that the manufacturer must keep sufficient inventories at the retailer’s site such that the retailer’s service level be unchanged. Furthermore, following LST and Cachon and Zipkin (1999), we assume that the retailer has relatively higher holding costs than the manufacturer, because additional costs (e.g., transportation costs) probably have been invested in the inventory at the retailer. The result of this assumption is that the manufacturer will want to keep all cycle stock at its own site and maintain only safety stock at the retailer’s location.

Because the demand structure and replenishment lead time remain unchanged, the average inventory at the retailer is

$$I_{t}^{VMI} = k\sigma \sqrt{v}.$$  

The average retailer’s inventory can be generalized to the case of VMI with replenishment of $g$ times per period as follows:

$$I_{mg}^{VMI} = \frac{1}{g} \frac{d}{2(1 - \rho)} + K\sigma \sqrt{v}, \quad \text{and}$$

$$I_{rg}^{VMI} = k\sigma \sqrt{v}.$$  

The benefits and benefit distribution ratio with VMI are:

$$\Delta I_{m}^{VMI} = I_{m} - I_{m}^{VMI} = \frac{g - 1}{g} \frac{d}{2(1 - \rho)} + K\sigma \left(\sqrt{V} - \sqrt{v}\right),$$

$$\Delta I_{t}^{VMI} = I_{t} - I_{t}^{VMI} = \frac{d}{2(1 - \rho)},$$

$$G_{\%}^{VMI} = \left[\frac{g - 1}{g} + \frac{2}{d}(1 - \rho)K\sigma \left(\sqrt{V} - \sqrt{v}\right)\right]\left[\frac{d + 2(1 - \rho)K\sigma \sqrt{v}}{d + 2(1 - \rho)K\sigma \sqrt{V}}\right].$$

Table 1 provides a summary of the modeling results.

<table>
<thead>
<tr>
<th>Summary of model results</th>
<th>$\Delta I_m$</th>
<th>$\Delta I_t$</th>
<th>$G_{%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS</td>
<td>$K\sigma \left(\sqrt{V} - \sqrt{v}\right)$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>CRP</td>
<td>$\frac{g - 1}{g} \frac{d}{2(1 - \rho)} + K\sigma \left(\sqrt{V} - \sqrt{v}\right)$</td>
<td>$\frac{g - 1}{g} \frac{d}{2(1 - \rho)}$</td>
<td>$\frac{[g - 1]d + 2g(1 - \rho)K\sigma \left(\sqrt{V} - \sqrt{v}\right)}{[g - 1]d + 2(1 - \rho)K\sigma \sqrt{v}}$</td>
</tr>
<tr>
<td>VMI</td>
<td>$\frac{g - 1}{g} \frac{d}{2(1 - \rho)} + K\sigma \left(\sqrt{V} - \sqrt{v}\right)$</td>
<td>$\frac{d}{2(1 - \rho)}$</td>
<td>$\frac{[g - 1]d + 2g(1 - \rho)K\sigma \left(\sqrt{V} - \sqrt{v}\right)}{[g - 1]d + 2(1 - \rho)K\sigma \sqrt{v}}$</td>
</tr>
</tbody>
</table>

IS, information sharing; CRP, continuous replenishment program; VMI, vendor managed inventory; $\Delta I_m$, changes in the manufacturer’s inventory from the base case; $\Delta I_t$, changes in the retailer’s inventory from the base case; $G_{\%}$, distribution ratio of the percentage in inventory reductions (manufacturer to retailer).
3.5. Analysis of results

In this section, we analyze the results obtained in the last section and present a number of propositions.

3.5.1. Benefits to the manufacturer

From the results in Table 1, we obtain the following propositions regarding benefits to the manufacturer:

**Proposition 1a.** For the same demand, lead time, and replenishment frequency parameters, the following inequality always holds true for $\rho \geq 0$:

$$0 \leq \Delta I_m^{\text{IS}} \leq \Delta I_m^{\text{CRP}} \leq \Delta I_m^{\text{VMI}}.$$

This proposition indicates that the manufacturer always enjoys non-negative benefits of inventory reduction from the implementation of any of the three collaborations with the retailer. With IS, the manufacturer’s safety stock is reduced because the manufacturer knows the end-consumer demand and demand variation in real time, instead of observing consumer demand through the retailer’s orders. In addition, CRP and VMI bring extra benefits to the manufacturer because CRP and VMI improve replenishment practices. As we show in Table 1, CRP and VMI also reduce cycle inventory. Furthermore, VMI may outperform CRP because it further lowers safety stock due to the changes in the ordering process.

**Proposition 1b.** For CRP and VMI, $\Delta I_m^{\text{CRP}}$ and $\Delta I_m^{\text{VMI}}$ have the following properties: (i) increase in $d$ (when $g > 1$), (ii) increase in $\rho$, (iii) increase in $g$, and (iv) increase in $K$.

This proposition shows that as mean demand and autocorrelation in demand increase, the manufacturer receives increasing inventory cost savings with the implementation of either CRP or VMI. More frequent replenishments between the manufacturer and retailer lead to greater inventory cost savings for the manufacturer. Furthermore, because $K$ is increasing in $P/(P + H)$, $\Delta I_m^{\text{CRP}}$ and $\Delta I_m^{\text{VMI}}$ are also increasing in $K$. Hence, when the stockout penalty cost, $P$, is high relative to the inventory holding cost, $H$, CRP and VMI lead to greater inventory reductions.

3.5.2. Benefits to the retailer

From the results in Table 1, we obtain the following propositions:

**Proposition 2a.** For the same demand, lead time, and replenishment frequency parameters, the following inequality always holds true:

$$0 = \Delta I_r^{\text{IS}} \leq \Delta I_r^{\text{CRP}} \leq \Delta I_r^{\text{VMI}}.$$

This proposition indicates that the retailer does not always receive benefits from an inventory reduction through the implementation of these collaborations. If only IS is implemented, the retailer’s inventory levels remain unchanged. The implementation of CRP and VMI reduce the retailer’s cycle inventory because of more frequent replenishments. When $g = 1$ (i.e., one replenishment per review period), the benefits from CRP to the retailer are also reduced to 0. When $g$ approaches positive infinity (i.e., continuous replenishment), the benefits from CRP to the retailer increase to $\frac{d}{2(1-\rho)}$, the same as with VMI.

**Proposition 2b.** For CRP and VMI, $\Delta I_r^{\text{CRP}}$ and $\Delta I_r^{\text{VMI}}$ have the following properties: (i) increase in $d$, (ii) increase in $\rho$, and (iii) increase in $g$ (for $\Delta I_r^{\text{CRP}}$ only).

This proposition shows that as mean demand and autocorrelation increase, the retailer receives greater inventory cost savings with the implementation of either CRP or VMI (compared with IS only). It also shows that with CRP, more frequent replenishments between the manufacturer and the retailer lead to greater inventory cost savings for the retailer. The proposition suggests that it is favorable for the retailer to adopt CRP or VMI, especially when demand is high or highly correlated with the previous period’s demand. Additional benefits from CRP may also be received if the retailer can persuade the manufacturer to replenish inventory as frequently as possible.

3.5.3. Benefit distribution

From Table 1, we obtain the following propositions regarding the benefit distribution between the manufacturer and the retailer:
Proposition 3a. For the same demand, lead time, and replenishment frequency parameters, the following inequalities always holds true: (i) \( G_{IS}^B > G_{CRP}^B \), (ii) \( G_{IS}^B > G_{VMI}^B \), and (iii) \( G_{CRP}^B \geq G_{VMI}^B \) when \((g - 1)\sqrt{V - g}\sqrt{V^2 + (g - 1)\sqrt{V^2} - \frac{g - 1}{2g(1 - \rho)\bar{\sigma}^2}}\).

This proposition indicates that the benefit distributions for IS, CRP, and VMI between the manufacturer and the retailer are not consistent. The manufacturer, relative to the retailer, receives a greater share of benefits from inventory reduction with IS than from CRP or VMI. Comparing VMI to CRP, the benefits are relatively more favorable to the retailer with VMI. The reason for this benefit shift from the manufacturer to the retailer is that VMI, unlike CRP, moves inventory from the retailer to the manufacturer because of the relatively lower inventory holding costs at the manufacturer’s location. However, as Proposition 1a suggests, the absolute benefits to the manufacturer are higher with VMI than with CRP, even though inventories at its locations may have increased.

Proposition 3b. Distribution ratios in percentages for both CRP and VMI have the following properties: (i) decrease in \( g \) for CRP and (ii) increase in \( g \) for VMI.

This proposition indicates that more frequent replenishments reduce the benefit distribution ratio under CRP, suggesting that the retailer gains more benefits from increasing replenishment frequency relative to the manufacturer. Stated differently, marginal benefits gained from increasing replenishment frequency under CRP favor the retailer over the manufacturer. However, under VMI, more frequent replenishments increase the benefit distribution ratio, implying that the manufacturer experiences greater gains relative to the retailer.

4. Numerical examples and discussion

In this section, we discuss the impacts of demand parameters and replenishment frequency on the value of CRP and VMI using numerical examples. These numerical findings highlight the managerial applications of our model and findings. For example, they show under which demand conditions CRP and VMI are most valuable and how logistics managers may strategically determine their replenishment frequency to obtain the greatest benefits from these programs. Finally, we discuss the relationship between our findings and previous literature.

4.1. Impact of demand parameters

The demand parameters refer to mean demand, \( d \); first-order autocorrelation in demand, \( \rho \); and the standard deviation of demand, \( \sigma \). We specify fixed values for all other parameters (see Table 2) and vary \( d \), \( \rho \), and \( \sigma \) to examine how these parameters affect the magnitude of inventory reductions for manufacturers and retailers.

To analyze the impact of demand, \( d \), on inventory levels, we set \( \rho = 0.7 \) and \( \sigma = 50 \) and vary \( d \) from 50 to 150. To analyze the impact of autocorrelation, \( \rho \), we set \( d = 100 \) and \( \sigma = 50 \) and vary \( \rho \) from 0.1 to 0.9. Finally, to analyze the impact of deviations in demand, \( \sigma \), we set \( d = 100 \) and \( \rho = 0.7 \) and vary \( \sigma \) from 10 to 100. The baseline case is the two-level supply chain without any collaboration; that is, without IS, CRP, or VMI.

Fig. 2 illustrates the results of the impact of \( d \) on the manufacturer’s and the retailer’s inventory reductions with CRP and VMI. We observe that inventory reductions for both the manufacturer and the retailer with CRP and VMI are increasing in \( d \) (see Propositions 1b and 2b). For example, when \( d = 50 \), the inventory reductions for the manufacturer and retailer with CRP (over the base case) are approximately 27% and

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Assumed values of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer</td>
<td>Retailer</td>
</tr>
<tr>
<td>Shortage costs ((P, \rho))</td>
<td>25</td>
</tr>
<tr>
<td>Holding costs ((H, h))</td>
<td>1</td>
</tr>
<tr>
<td>Lead time ((L, l))</td>
<td>5</td>
</tr>
<tr>
<td>Replenishments per period ((g))</td>
<td>N/A</td>
</tr>
</tbody>
</table>
8%, respectively. At the higher value of demand, \( d = 150 \), the inventory reductions increase to approximately 32% and 18%, respectively. The figure also shows that the retailer’s inventory position is affected by the magnitude of demand \( (d) \) to a greater degree than is the manufacturer’s inventory position. This result suggests that for both CRP and VMI, the retailer achieves greater benefits from increasing demand than does the manufacturer.

Fig. 3 reports the results of the impact of autocorrelation, \( \rho \), on the manufacturer’s and the retailer’s inventory reductions with CRP and VMI. We observe that inventory reductions for both the manufacturer and the retailer are increasing in \( \rho \) (see Propositions 1b and 2b), which suggests that collaborations, such as CRP or VMI, are especially beneficial to the participants in the case of highly autocorrelated demand. Intuitively, the manufacturer and the retailer must keep high inventory levels to meet service level requirements when demand is highly autocorrelated. Thus, CRP and VMI can provide greater inventory reduction benefits when \( \rho \) is high. The figure also shows that the inventory reduction benefits from VMI are always at least as great as those from CRP for both the manufacturer and the retailer.

Fig. 4 reports the results of the impact of the standard deviation in demand, \( \sigma \), on the manufacturer’s and the retailer’s inventory reductions with CRP and VMI. We observe that inventory reductions for both the manufacturer and the retailer with CRP and with VMI are decreasing in \( \sigma \). This result suggests that as demands become relatively more predictable the benefits from using programs such as CRP and VMI, decrease.
These numerical examples show the sensitivity of inventory reductions to changes in the demand parameters. The results suggest that CRP and VMI may be more beneficial in some demand situations, such as when demand and/or autocorrelation is high, than in others. In practice, different types of products possess different patterns of demand. Our results provide managers with insights into which types of products are more suited for CRP and VMI programs. Hence, by carefully choosing the products to be managed by CRP or VMI, managers may be able to increase the program’s success rate.

4.2. Replenishment Frequency in CRP and VMI

Fig. 5 reports the results of the impact of $g$ on the manufacturer’s and the retailer’s inventory reductions with CRP and with VMI. We set the demand process parameters as follows: $d = 100$, $\rho = 0.7$, and $\sigma = 50$, and we vary the replenishment frequency parameter, $g$, from 1 to 10. All other parameters follow the values defined in Table 2.

We observe that inventory reductions for both the manufacturer and the retailer with CRP are increasing in $g$, with diminishing slopes. Inventories for the retailer with VMI remain constant, whereas inventory reductions for the manufacturer are increasing in $g$. This result suggests that for all cases, except for the retailer with VMI, more frequent replenishments result in lower average inventories. This result makes intuitive sense, in
that for a level of demand, we might expect that a firm with daily replenishments carries, on average, less stock than a firm with weekly replenishments. However, for the retailer with VMI, inventory does not decrease because cycle inventory is strategically maintained at zero at the retailer’s site, regardless of replenishment frequency, so that increasing replenishments does not affect the retailer’s average inventory level.

Fig. 6 shows the impact of replenishment frequencies, \( g \), on the CRP and VMI benefit distribution ratios (in percentage terms, \( G_{nu} \)) between manufacturers and retailers. The demand process parameters are fixed at \( d = 100, \rho = 0.7, \) and \( \sigma = 50 \), whereas the replenishment frequency parameter, \( g \), is allowed to vary from 2 to 10 times per order cycle. All other parameters follow the values defined in Table 2. We observe that the distribution ratio is decreasing in \( g \) under CRP and increasing in \( g \) under VMI. This result suggests that marginal benefits from increasing replenishment frequencies are distributed unevenly between the manufacturer and the retailer and inconsistently between CRP and VMI. Especially as replenishments increase, CRP is more favorable to the retailer, whereas VMI, with ordering processes managed by the manufacturer, is more favorable to the manufacturer. Note that both distribution ratios are greater than 1, indicating that the manufacturer always receives relatively more benefits than the retailer.

These numerical examples show how inventory reductions change with replenishment frequency. Replenishment frequency is a decision variable with a significant impact on the benefits and benefit distribution of CRP and VMI. Our findings provide managers with insights into how to determine the replenishment frequency to best exploit the potentials of CRP and VMI, especially when the benefits to the manufacturer and the retailer are not congruent in terms of replenishment frequency. With VMI, the manufacturer may bargain for a relatively higher replenishment frequency, whereas with CRP, the retailer may bargain for a relatively higher replenishment frequency.

4.3. Contributions to the literature

In this paper, we construct an analytical model to examine the benefits, in terms of inventory reductions, and the distribution of these benefits derived from IS, CRP, and VMI. Results from the model show that positive benefits may be generated from implementing any of these supply chain coordination programs. Both CRP and VMI aim to reduce a participant’s inventory by increasing replenishment frequency. With more frequent replenishments, both the manufacturer and the retailer may reduce cycle stocks. However, VMI differs from CRP in that it reduces both the manufacturer’s cycle inventory and its safety stock.

Results from the model show that benefits from inventory reduction are not equally distributed between participants with VMI. The distribution of benefits is determined by parameters, such as replenishment frequency and inventory holding costs. The analysis suggests that the upstream participant is likely to receive relatively greater benefits than the downstream participant from VMI.

The results of this paper are consistent with previous research, such as Waller et al. (1999) regarding VMI, Daugherty et al. (1999) for ARP, Stank et al. (1999) with ECR, Premkumar (2000) regarding
inter-organizational systems and extranets, Raghunathan (1999) in terms of CFAR, Strader et al. (1999) with regards to extranets, LST regarding CRP, and Choudhury et al. (1998) regarding electronic markets. Previous research has shown that the implementation of interorganizational systems, in general, can benefit the supply chain by reducing costs. Our study provides further evidence of the conditions for inventory cost reduction that can be achieved through the implementation of IOSs such as CRP and VMI.

Both LST and Xu et al. (2001) conclude that positive inventory cost savings result from information sharing in a supply chain. Their approaches focus on how information sharing can reduce forecasted demand variation (i.e., the bullwhip effect) and thereby achieve better supply chain coordination. Cachon and Zipkin (1999) use game theory modeling in a two-stage supply chain to demonstrate that a cooperative inventory policy is better than a competitive one. RY calculate the benefits of CRP to the manufacturer and the retailer under AR(1) demand. Our results, derived from a model similar to that used by LST and RY, also demonstrate that inventory cost savings can be achieved through collaboration, whether CRP and VMI. We further examine how those savings are distributed between participants and show how the type of program can affect the distribution of benefits between manufacturers and retailers.

5. Concluding remarks

In this paper, we study the value of IS, CRP, and VMI by constructing and analyzing an analytical model. Our model shows that these supply chain programs may bring benefits in terms of inventory reduction to the participants. The magnitude of the benefits is determined by the demand process and logistics parameters. In addition, the benefits may not be equally distributed between participants. The distribution of these benefits is determined by both the type of program in place and the demand process and logistics parameters. In particular, the use of VMI and CRP tend to result in greater inventory reductions for both the manufacturer and the retailer than does IS alone. The extent of these inventory reductions increases as mean demand increases and as the autocorrelation of demand increases. Inventory reductions increase, as well, as replenishment frequency rises for both CRP and VMI. However, the benefits tend to favor the manufacturer under VMI and the retailer under CRP as replenishment frequency increases.

Relevant literature consistently recognizes that inventory reductions can be achieved by implementing initiatives such as IS, CRP, or VMI. This paper contributes to the literature by constructing an analytical model to provide a better understanding of the extent and distribution of inventory cost savings. Furthermore, this research contributes to the IOS theories by introducing a number of new instruments that can be used to measure the business value of IOSs. These new instruments include inventory holding cost, replenishment frequency, and fixed replenishment costs. The research also articulates the effects of the interaction between information sharing and other supply chain coordination initiatives, thereby providing a better understanding of the value of information sharing in supply chains.

In addition, this research provides practitioners with a useful framework for evaluating potential benefits within various interorganizational systems. Our models indicate how important supply chain parameters affect both the size and the distribution of expected supply chain benefits. Managers can examine their own operations in light of these findings to determine which of the interorganizational systems may be most beneficial to their firms. They may also strategically choose the set of products to be managed through CRP and VMI and select a favorable replenishment frequency for improved benefit realization. Finally, managers may be able to use this framework to determine whether side payments may be appropriate or necessary for the implementation of IS, CRP, or VMI. However, the results should be applied with caution, because additional costs, such as operating and implementing costs, are incurred with CRP and VMI, and system trade-offs, including customer service level and operation flexibility, may not be captured fully by the analytical model.

Although we believe that our model is representative of many actual supply chains, we recognize the results are limited due to the settings assumed in the model. For example, demand is assumed to be an AR(1) process. Although the AR(1) process represents demand for many products in specific industries (LST), it does not apply to firms that have higher-order AR demand processes or nonstationary demand, such as seasonal fashion apparel firms. In addition, our assumption that the manufacturer does not use historical order information to forecast the retailer’s order quantity limits the generalizability of our findings about the value of information.
sharing. This assumption, however, does not affect our findings regarding the value of CRP and VMI. Further research might center on extending the current model by relaxing these assumptions. Whereas this research assumes that the inventory review period does not change but replenishment increases to $g$ times per review period, additional research should also extend the model to allow inventory review period changes. Another limitation of our model that can be extended in future research is that we assume backorders at the store level. Often times, retailer face lost sales, instead of backorders. Finally, we acknowledge that another possible main benefit of VMI that we did not assess is that suppliers can smooth their own demands, reducing their out of stocks and therefore reducing the lead time variability.

Appendix

**Lemma 1.** Suppose a period $t$ is divisible and equally divided into $n$ periods. Demand during the period $t$ follows $\text{AR}(1)$, and the demand for $\forall n$ period also follows $\text{AR}(1)$; that is, $D_t = d + \rho D_{t-1} + \epsilon_t$ and $D'_t = d' + \rho' D'_{t-1} + \epsilon'_t$, where $t = nt'$ (e.g., $t$ may be a week, and $t'$ may be a day, then $n$ is 7 in this case). Then, the demand parameters have the following relationships:

$$D_t = \frac{1 - \rho^t}{n(1 - \rho)} d + \rho^t D_{t-1} + \epsilon_t,$$

where $n = 1, 2, \ldots, t' = tm, \ d > 0, -1 < \rho < 1$, and $\epsilon_t$ is i.i.d. normally distributed with mean zero and variance:

$$\sigma^2 = \left( \sum_{i=1}^{n-1} (1 + \rho^{i-t})^2 \cdot \left( \sum_{j=1}^{i} \rho^{j-1} \right)^2 + \left( \sum_{i=1}^{n} \rho^{i-1} \right)^2 \right)^{-1},$$

**Proof for Lemma 1.** Because period $t$ includes $nt'$ periods, that is, $t' + 1, t' + 2, \ldots, t' + n$, $D_t$ can be expressed as:

$$D_t = \sum_{i=1}^{n} D_{t+i}$$

$$= \frac{1}{1 - \rho'} \left[ d' \sum_{i=1}^{n} (1 - \rho^i) + \rho' (1 - \rho^n) D_t \right] + \epsilon_{t+n} + (1 + \rho') \epsilon_{t+n-1} + \cdots + (1 + \rho' + \cdots + \rho^{n-1}) \epsilon_{t+1}. \tag{A-1}$$

Similar to $D_{t-1}$,

$$D_{t-1} = \sum_{i=0}^{n-1} D_{t-i}$$

$$= \frac{1}{1 - \rho'} \left[ d' \sum_{i=1}^{n} (1 - \rho^i) + \rho' (1 - \rho^n) D_{t-1} \right] + \epsilon_{t} + (1 + \rho) \epsilon_{t-1} + \cdots + (1 + \rho + \cdots + \rho^{n-1}) \epsilon_{t-n+1}. \tag{A-2}$$

Using the recursive relationship of $D_t$, we can obtain the relationship between $D_t$ and $D_{t-n}: D_t = d' \frac{1 - \rho}{1 - \rho'} + \rho'^n D_{t-n} + \left( \rho'^{n-1} \epsilon_{t-n+1} + \rho'^{n-2} \epsilon_{t-n+2} + \cdots + \epsilon_t \right). \tag{A-3}$$

Substituting expression (A-3) into the expression (A-1), we can rewrite it as:

$$D_t = \frac{1}{1 - \rho'} \left[ d' \sum_{i=1}^{n} (1 - \rho^i) + d' \rho' (1 - \rho^n) (1 - \rho') \right] + \rho' (1 - \rho^n) \rho^n D_{t-n}$$

$$+ \rho' (1 - \rho^n) \left( \rho'^{n-1} \epsilon_{t-n+1} + \rho'^{n-2} \epsilon_{t-n+2} + \cdots + \epsilon_t \right) + \epsilon_{t+n} + (1 + \rho') \epsilon_{t+n-1}$$

$$+ \cdots + (1 + \rho' + \cdots + \rho^{n-1}) \epsilon_{t+1}. \tag{A-4}$$
Because $D_t$ and $D_{t-1}$ have the relationship, $D_t = d + \rho D_{t-1} + \epsilon_t$, substituting the expression $D_{t-1}$ in A-2 into the assumed relationship, we obtain $D_t$ expressed in terms of $D_{t-n}$ as:

$$
D_t = d + \frac{\rho}{1 - \rho^2} \left[ d' \sum_{i=1}^{n} (1 - \rho^n) + \rho' (1 - \rho^n) D_{t-n} \right] \\
+ \rho \left[ \epsilon_t + (1 + \rho') \epsilon_{t-1} + \cdots + (1 + \rho' + \cdots + \rho^n) \epsilon_{t-n-1} \right] + \epsilon_t.
$$

(A-5)

Comparing the coefficients for each term for the two $D_t$ equations, (A-4) and (A-5), and collecting terms, we have the following unique solutions:

$$
\rho' = \rho^\frac{1}{2},
$$

(A-6)

$$
d' = \frac{1 - \rho^\frac{1}{2}}{n(1 - \rho)} d,
$$

(A-7)

$$
\sigma^2 = \left[ \sum_{i=1}^{n-1} (1 + (\rho^{n-i})^2) \left( \sum_{i=1}^{\lceil n/2 \rceil} \rho^{i-1} \right)^2 + \left( \sum_{i=1}^{\lceil n/2 \rceil} \rho^{i-1} \right)^2 \right]^{-1} \sigma^2.
$$

(A-8)

Eqs. (A-6)–(A-8) define the corresponding changes of coefficients in the AR(1) process between time series $t'$ and $t$. □

**Lemma 2.** Suppose a period $t$ is divisible and equally divided into $n$ period; the average demand for a $1/n$ period is just $1/n$ times the average demand for a whole $t$ period. That is:

$$
\text{Average } (D_t) = \frac{1}{n} \text{Average } (D_1), \text{ where } t \text{ includes } n \text{ periods of } t'.
$$

**Proof for Lemma 2.** From Lemma 1, we learn that the demand for each subdivided period, $1/n$ period, can be expressed as follows: $D_t = \frac{1 - \rho}{g(1 - \rho)} d + \rho D_{t-1} + \epsilon_t$, where $t' = 1/n$ period. The average demand for each $t'$ period (i.e., $1/n$ period) when $t'$ approaches positive infinity is $\lim_{t' \to \infty} D_t = \frac{1}{n} \frac{1 - \rho}{1 - \rho^2} d$. The average demand for each $t$ period when $t$ approaches positive infinity is $\lim_{t \to \infty} D_t = \frac{1}{n} \frac{1 - \rho}{1 - \rho^2} d$. Therefore, the average demand for a $1/n$ period is just $1/n$ times average demand for a whole $t$ period. □

**Proof for Proposition 1a.** It is obvious that $\Delta l_{\text{IS}}^m \leq \Delta l_{\text{CRP}}^m$, because $\frac{1 - \rho}{g(1 - \rho)} d$ is always greater than or equal to zero. $\frac{1 - \rho}{g(1 - \rho)} d$ equals zero when $g$ equals 1, which is the case with IS, a special case of CRP. According to LST and Eq. (5), $\Delta l_{\text{IS}}^m$ is always greater than or equal to zero because $\sqrt{V'} \leq \sqrt{V}$. The relationship between $\Delta l_{\text{CRP}}^m$ and $\Delta l_{\text{VMI}}^m$ is determined by the relationship between $\sqrt{V'}$ and $\sqrt{V}$. From $V' = \frac{1}{(1 - \rho^2)} \left\{ (1 - \rho^2)^2 + \sum_{i=1}^{L} (1 - \rho^{L+i+1}) \right\}$, we can determine $V'$ is a sum of $(L + 1)$ expressions $\frac{1}{(1 - \rho^2)}$, which is a common factor for both $\sqrt{V'}$ and $\sqrt{V}$ and therefore is not included, such that $(1 - \rho^{L+2})^2$, $(1 - \rho^{L+3})^2$, $(1 - \rho^{L+4})^2$, $(1 - \rho^{L+5})^2$, …, $(1 - \rho^{L+L})^2$. From $V = \frac{1}{(1 - \rho^2)} \sum_{i=1}^{L+1} (1 - \rho^i)^2$, we find that $V'$ is also a sum of $(L + 1)$ expressions, such that $(1 - \rho)^2$, $(1 - \rho^2)^2$, $(1 - \rho^3)^2$, …, $(1 - \rho^{L})^2$. For $0 \leq \rho < 1$, $\rho^n \leq \rho^m$ when $n > m$, so $(1 - \rho^n)^2 \geq (1 - \rho^m)^2$. Comparing the $L + 1$ terms in $V'$ one by one with corresponding terms in $V'$ demonstrates that each term in $V'$ is smaller than the corresponding term in $V$; that is, $(1 - \rho)^2 \leq (1 - \rho^{m+2})^2$, $(1 - \rho^2)^2 \leq (1 - \rho^{m+3})^2$, …, $(1 - \rho^L)^2 \leq (1 - \rho^{L+2})^2$. Therefore, we can conclude that $V' \leq V$, and then, $\Delta l_{\text{CRP}}^m \leq \Delta l_{\text{VMI}}^m$ when $\rho \geq 0$. □

**Proof for Proposition 3a.** It is obvious that $G_{\text{IS}}^m$ is always greater than or equal to $G_{\text{CRP}}^m$, because $G_{\text{IS}}^m$ equals positive infinity. When $g = 1$, $G_{\text{IS}}^m$ also equals infinity. Similarly, $G_{\text{GR}}^m$ is always greater than $G_{\text{VMI}}^m$. However, $G_{\text{VMI}}^m$ does not approach positive infinity for any $g$. For the relationship between $G_{\text{CRP}}^m$ and $G_{\text{VMI}}^m$, by recollecting the terms in the expressions, we can obtain:
\[
G_{\text{CRP}}^{\%} = \frac{(g - 1)d + 2g(1 - \rho)K\sigma\sqrt{V - \sqrt{V^2}}}{(g - 1)d} \left[ \frac{d + 2(1 - \rho)k\sigma\sqrt{V}}{d + 2(1 - \rho)K\sigma\sqrt{V}} \right] \frac{d + 2(1 - \rho)k\sigma\sqrt{v}}{d + 2(1 - \rho)K\sigma\sqrt{v}}.
\]

\[
G_{\text{VMI}}^{\%} = \left[ \frac{g - 1}{g} \right] + \frac{2}{d(1 - \rho)K\sigma\sqrt{V - \sqrt{V^2}}} \left[ \frac{d + 2(1 - \rho)k\sigma\sqrt{v}}{d + 2(1 - \rho)K\sigma\sqrt{v}} \right].
\]

We let \(G_{\text{CRP}}^{\%} - G_{\text{VMI}}^{\%} = \left( \frac{1}{g} + \frac{2(1 - \rho)K\sigma}{\sqrt{V - g\sqrt{V^2} + (g - 1)\sqrt{V^2}}} \right)B_1 \geq 0\), where \(B_1 = \frac{d + 2(1 - \rho)k\sigma\sqrt{v}}{d + 2(1 - \rho)K\sigma\sqrt{v}}\). By collecting terms, we can obtain the conditions in which \(G_{\text{CRP}}^{\%}\) is greater than \(G_{\text{VMI}}^{\%}\): \(\sqrt{V - g\sqrt{V^2} + (g - 1)\sqrt{V^2}} \geq \frac{g - 1}{g - 1} - \frac{1}{g - 1}(1 - \rho)\). \(\Box\)

**Proof for Proposition 3b.** For CRP, \(G_{\text{CRP}}^{\%} = \left[ \frac{1}{g} + \frac{2(1 - \rho)K\sigma\sqrt{V - \sqrt{V^2}}}{d} \right] \left[ \frac{d + 2(1 - \rho)k\sigma\sqrt{v}}{d + 2(1 - \rho)K\sigma\sqrt{v}} \right] \). So we can get: \(\frac{\partial G_{\text{CRP}}^{\%}}{\partial g} = B \cdot \left( - \frac{1}{(g - 1)} \right) < 0\), where \(B = \left[ \frac{2(1 - \rho)K\sigma\sqrt{V - \sqrt{V^2}}}{d} \right] \left[ \frac{d + 2(1 - \rho)k\sigma\sqrt{v}}{d + 2(1 - \rho)K\sigma\sqrt{v}} \right] > 0\). Therefore, \(G_{\text{CRP}}^{\%}\) is decreasing in \(g\).

Similarly, for VMI, \(G_{\text{VMI}}^{\%} = \left[ \frac{g - 1}{g} - \frac{2(1 - \rho)K\sigma\sqrt{V - \sqrt{V^2}}}{d} \right] \left[ \frac{d + 2(1 - \rho)k\sigma\sqrt{v}}{d + 2(1 - \rho)K\sigma\sqrt{v}} \right] \). So we can get: \(\frac{\partial G_{\text{VMI}}^{\%}}{\partial g} = \frac{1}{g} - \frac{1}{g} > 0\). Therefore, \(G_{\text{VMI}}^{\%}\) is increasing in \(g\). \(\Box\)

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